Probability: Random Variables

Expectation of Discrete Random Variables

We have two kinds of means, one for data and second for random variables. Suppose we roll a dice twice and get 5 and 3. This is our data, and the average is simply $\bar{x} = \frac{5+3}{2} = 4$. The possible outcomes of a roll is $1, 2, 3, 4, 5$ and 6; let's represent it as a discrete random variable:

$$X = \text{face value of the die toss}$$

Since every side is equally likely to occur we have:

$$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6) = \frac{1}{6}$$

We can put these values in the table:

Value of X	1	2	3	4	5	6
Prob	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 1: Probability Table for the Die Problem

Now the question is, what is the **expected value** or average of X? We get it using a simple formula:

$$E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = 3.5$$

More generally we can write

$$E(X) = x_1 \times P(X = x_1) + x_2 \times P(X = x_2) + \cdots + x_6 \times P(X = x_6)$$

Where in our example $x_1 = 1, x_2 = 2, , x_6 = 6$.

So to get expected value of a discrete random variable we average the values this random variable can be, and we average them by their probabilities. The values that occur with greater probability get higher weights in the average.

Generally, an expected value of a discrete random variable X that can take k different values $x_1, x_2, , x_k$ with probabilities $P(X = x_1), P(X = x_2), , P(X = x_k)$ has this formula:

$$E(X) = x_1 P(X = x_1) + x_2 P(X = x_2) + \cdots + x_k P(X = x_k) = \sum_{i=1}^{i=k} x_i P(X = x_i)$$

In our dice example $k = 6$, $x_1 = 1, x_2 = 2, , x_6 = 6$ with $P(X = x_1) = P(X = x_2) = \cdots = P(X = x_6) = 1/6$.