Probability: Random Variables

Using a Normal Probability Table

In this document we show how to use Normal Probability Tables which will be used very frequently in this course. We know that to find probabilities for continues random variables, we need to compute areas under their probability density functions. Here we focus on normal distribution which is of course continuous.

Example 1

Consider a standard normal distribution with mean 0 and standard deviation 1 (N(0,1)). The density function for this distribution looks like that:

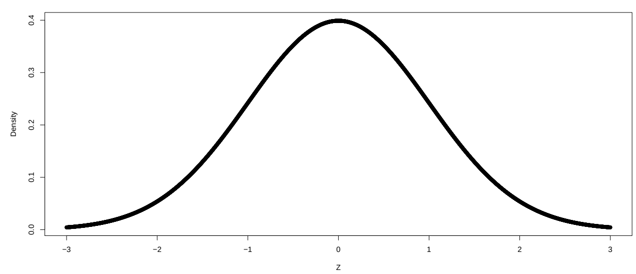


Figure 1: Density function of Normal(0,1)

Let Z be a random variable with this distribution. If we need to compute $P(Z \geq 1)$ (probability that Z is greater than 1), then we have to find area after 1 under the density function.

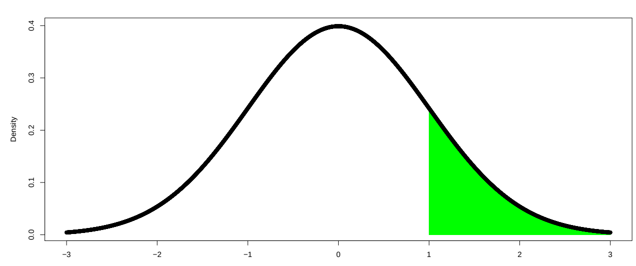


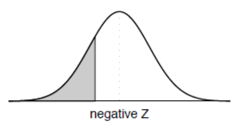
Figure 2: Probability of $Z \geq 1$

There is no closed form equation to find such area, but we have a standard normal probability table that we will use. Note that this table is for normal distribution with mean 0 ($\mu = 0$) and standard deviation 1 ($\sigma = 1$). In the next notes we will show how to find areas for normal distributions with different means and standard deviations. Here we assume that we are dealing with Normal(0,1) and call the random variable with this distribution Z. Notationaly we write

$$Z \sim N(0, 1)$$

We are using a table from `www.openintro.org/stat.textbook.php` and it is freely available. The normal probability table has two pages, for positive values of Z and negative values. Let's first look at negative values. The beginning of the table look like that:

Normal probability table



Second decimal place of Z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	Z
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0

Figure 3: Standard normal probability table for negative values of Z

As shown in the diagram, this table shows probabilities of being **less** than the relevant value. Since the mean of distribution is 0, then the Z values on this page are on the left half of the graph. The middle part of this table corresponds to probabilities while column and row heading are the Z values.

EXAMPLE 2

Suppose we are seeking probability that Z is less than -1.82 ($P(Z < -1.82)$). We first find a row corresponding to -1.8, the second decimal place corresponds to the column 0.02. Hence we see that $P(Z < -1.82) = 0.0344$. The table only gives us two decimal places, so any time you need to use a table, you'll have to round to 2 decimal places.

Suppose now we want $P(Z \leq -1.82)$. Since Z is a continuous random variable, the probability of it being any one particular number is 0. Therefore $P(Z = -1.82) = 0$ and $P(Z \leq -1.82) = P(Z < 1.82)$ so it is also 0.0344.

EXAMPLE 3

Now suppose we need the $P(Z > -1.82)$, this would be the area to the right of -1.82.

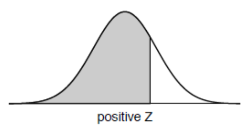
Since the total area under any density curve is 1 and using results from previous example we get,

$$P(Z > -1.82) = 1 - P(Z \leq -1.82) = 1 - 0.0344 = 0.9656$$

Note that the smallest Z value in the table is -3.49, and the probability of being less than this is very small, only 0.0002. For any smaller (more negative) value, the probability of being less than it is so small that we can approximate it by 0. So for example $P(Z < 4) \approx 0$.

The second page of the normal probability table works the same way, but for positive values of Z. The table looks like that:

Normal probability table



	Second decimal place of Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879

Figure 4: Standard normal probability table for positive values of Z

Since all the positive values of Z are greater than the mean, all of the probabilities in this page are **greater** than 0.5 as the area less than a positive Z covers more than half of the area under the curve. Otherwise, this side of the table works the same way.
Because of the symmetry of the normal distribution, we don't really need both pages. For example, $P(Z < -1.82)$ is the same as the $P(Z > 1.82)$. To verify this, observe that $P(Z \leq 1.82) = 0.9656$ (second page), and we get:

$$P(Z > 1.82) = 1 - P(Z \leq 1.82) = 1 - 0.9656 = 0.0344 = P(Z < -1.82)$$

Example 4

Suppose now we want to find the probability between two Z values, for example $P(-3.12 < Z < 0.14)$. To get this, we need to remember that the table only gives us probabilities of being less than a value. But the area under the curve between -3.12 and 0.14 is the same as the area to the left of 0.14 minus the area to the left of -3.12:

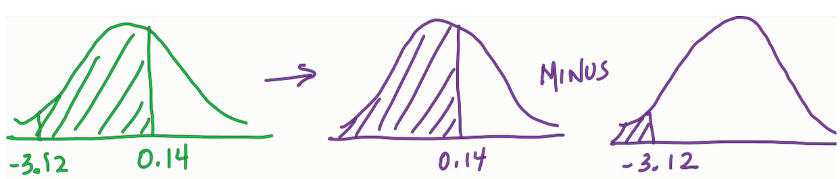


Figure 5: Probability between -3.12 and 0.14

So we get

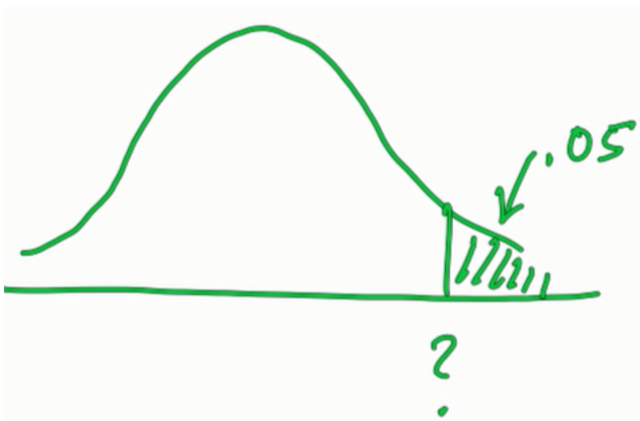
$$P(-3.12 < Z < 0.14) = P(Z < 0.14) - P(Z < -3.12) = 0.5557 - 0.0009 = 0.5548$$

The first number above we get from the positive Z page while the second number from the negative Z page of the table.

There is one more way to use the normal probability table. Suppose we know the probability, and want to find a corresponding value of Z.

Example 5
What's the value from the standard normal distribution that has probability less than it of 0.10? This probability is less than one-half, so we know we need the negative Z page of the table. Scanning through the probabilities, the probability closest to 0.10 is 0.1003, corresponding to a Z value of -1.28. So, to an accuracy of 2 decimal places, we know that -1.28 is the value from a standard normal distribution such that the probability of being less than it is 0.10 ($P(Z < -1.28) \approx 0.10$).

Now suppose we wanted the value such that the probability of being greater than it is 0.05.



For this value, the probability of being less than it would be 0.95. Scanning the positive Z page of the table, 0.95 is half way between two values, corresponding to a Z of 1.64 and 1.65. Since we don't have a clear choice for the closer value, we will take the value of Z right in the middle, so we estimate that 1.645 as the value from a standard normal distribution with probability above it of 0.05 ($P(Z > 1.645) \approx 0.05$).

Many random variables have normal distributions, but few naturally have standard normal distributions. However any normal random variable can be transformed to a standard normal random variable by subtracting the mean and dividing the result by the standard deviation which then allows us to use the standard normal probability table, and solve lots of problems involving the normal distribution.