# Probability: Events

## Conditional Probability and Tree Diagrams

In this document we show how to use a tree diagram to calculate conditional probabilities.

Example:
Suppose we are given the following (hypothetical) information:

- *First born children have a 50% chance of being female.*
- *If the first child is a girl then the probability that the second child is a girl is* $\frac{1}{3}$.
- *If the first child is a boy then the probability that the second child is a girl is* 0.40.

In this situation, what is the probability that the first child is a female if the second child is a female?

One convenient way to solve this problem is by drawing a tree diagram.
Since we know that *first born children have a 50% chance of being female* we draw two branches corresponding to females and males and add probabilities for these events (0.5 and 0.5).

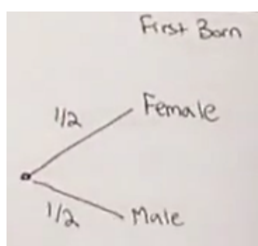


Figure 1: First part of the tree diagram

Next we are given that *if the first child is a girl then the probability that the second child is a girl is* $\frac{1}{3}$. This is information about the 'female' branch of the tree, and so we add another two branches to 'first born female' corresponding to female and male and write probabilities ($1/3$ for girl given girl and $1 - 1/3 = 2/3$ for boy given girl). The last piece of information is that *if the first child is a boy then the probability that the second child is a girl is* 0.40. Here we add two branches to 'first born male' and add probabilities (0.40 for girl given boy and $1 - 0.40 = 0.60$ for boy given boy).

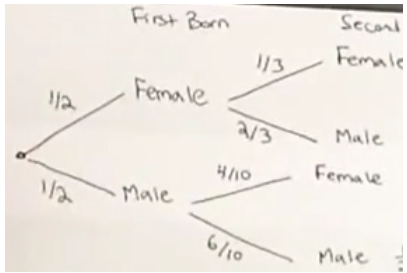


Figure 2: Completed tree diagram

The goal is to calculate:

$$P(\text{first female}|\text{second female}) = ?$$

Using the definition for conditional probability we get:

$$P(\text{first female}|\text{second female}) = \frac{P(\text{first female and second female})}{P(\text{second female})}$$

We can use the tree diagram to find the numerator and the denominator. To get $P(\text{first female and second female})$ we just multiply probabilities of 'first born female' and 'second female given first female'[1] which is $\frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$. Similarly $P(\text{first female and second male}) = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$.

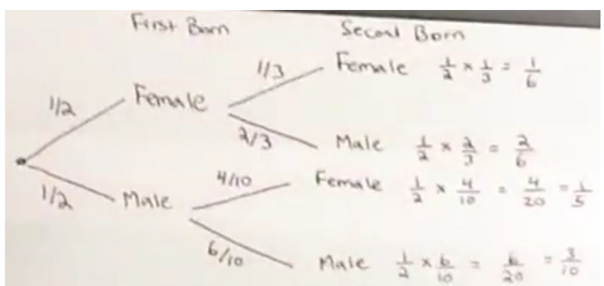


Figure 3: Tree diagram with 'and' probabilities

So far we have found that:

$$P(\text{first female and second female}) = \frac{1}{6}$$

To get $P(\text{second female})$ note that $P(\text{second female}) = P(\text{first female and second female}) + P(\text{first male and second female}) = \frac{1}{6} + \frac{1}{5} = \frac{11}{30}$.[2] Now we get the answer

$$P(\text{first female}|\text{second female}) = \frac{\frac{1}{6}}{\frac{11}{30}} = \frac{30}{66} \approx 0.45$$

[1] This is an application of the multiplication rule: $P(\text{B and A}) = P(\text{A}|\text{B}) \times P(\text{B})$

[2] This is an application of the addition rule for disjoint events.