Probability: Events

Statistical Independence and Multiplication Rules

We start with a definition.
Two events are **independent** if:

$$P(\text{A}|\text{B}) = P(\text{A})$$

The definition shows that event B does not have an effect on the probability of event A. Also, if events A and are independent then

$$P(\text{B}|\text{A}) = P(\text{B})$$

So, if two events are independent and using the definition for conditional probability we get:

$$P(\text{A}|\text{B}) = P(\text{A}) = \frac{P(\text{A and B})}{P(\text{B})} \text{ if } P(\text{B}) > 0$$

Rearranging the equation above we see that A and B are independent if

$$P(\text{A}) \times P(\text{B}) = P(\text{A and B})$$

This is a convenient way to check whether two events are independent or not.

Example
Suppose $P(\text{A}) = 0.3$, $P(\text{B}) = 0.1$ and $P(\text{A and B}) = 0.01$. *Are A and B independent?*
To check this we first calculate the product of the probabilities of the two events:

$$P(\text{A}) \times P(\text{B}) = 0.1 \times 0.3 = 0.03$$

It does not equal to $P(\text{A and B}) = 0.01$.
Thus A and B are **not** independent.