Probability: Events

Conditional Probability

In these notes we introduce conditional probability.

Suppose we want to toss a coin four times and on the first toss we get tails (T). *Given that a coin came up T on the first toss, what is the probability that we get at least 2 heads (H) out of four tosses?*
This is an example of conditional probability calculation, since we use some partial information (T on the first toss) to answer the question.

Example
Suppose we flip a coin 3 times. In this case we have 8 possible outcomes:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

What is the probability of obtaining at least two heads in three coin tosses?
First, event B represents outcomes with two or more heads:

$$\text{B} = \{\text{two or more heads}\}$$

Out of 8 outcomes we have 4 in event B, these are $\{HHH, HHT, HTH, THH\}$. Now since the probability of each outcome is 1/8 we get:

$$P(\text{B}) = 4 \times \frac{1}{8} = \frac{4}{8} = 0.5$$

Now suppose we have some partial information concerning the event. For example suppose we learn that the first toss is H. We explore how this information affects probability of event B, more precisely we need to find probability of getting at least 2 heads *given* that the first toss came up a H.

As before we start with two events:

$$\text{A} = \{\text{first toss is a head}\}$$

$$\text{B} = \{\text{two or more heads}\}$$

Since we know that the first toss is a H we can eliminate all the outcomes for which the first toss is a T. So now the possible outcomes are $\{HHH, HHT, HTH, HTT\}$. Among these outcomes, 3 of them have at least 2 heads. The notation for *Conditional probability of B given A* is $P(\mathrm{B}|\mathrm{A})$, and therefore we get:

$$P(\mathrm{B}|\mathrm{A}) = 3 \times \frac{1}{4} = \frac{3}{4}$$

This means that probability of getting at least 2 heads in 3 tosses given that the first toss is a head is 0.75.

The general definition of conditional probability is given by:

$$P(\mathrm{B}|\mathrm{A}) = \frac{P(\mathrm{A\ and\ B})}{P(\mathrm{A})}, \quad P(\mathrm{A}) > 0$$

Here event A gives us partial information about event of interest which is B.